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A theoretical approach for the thermal expansion behavior of the particulate reinforced aluminum matrix composite

Part I *A thermal expansion model for composites with mono-dispersed spherical particles*

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Microstructural observation revealed that the increase in the volume fraction of SiC particles lowers the coefficient of thermal expansion (CTE) of the composite, and the CTE of the metal matrix composites is proportional to the size of the Si phase. To analyze the thermal expansion behavior of aluminum matrix composites, a new model for the CTE of the mono-dispersed binary composite on the basis of Ashelby's cutting and welding approach was proposed. In the theoretical model, it was considered that during cooling relaxation of residual stresses could create an elasto-plastic deformation zone around a SiC or Al_2O_3 particle in the matrix. The size of reinforced particles and other metallurgical factors of the matrix alloy and composite were also considered. In this model, the interacting effect between the reinforced hard particle and the soft matrix is considered by introducing the influence of the elasto-plastic deformation zone around a particle, which is distinguished from the previous models. It was revealed that the CTE of the composite are influenced by the particle volume fraction, the elastic modulus and Poisson's ratio as well as the elasto-plastic deformation zone size and the particle size.

Nomenclature

a	constant $= r_p/r$
b	Burger's vector
C	constant
E_m	Young's modulus
G_m	the shear modulus of matrix
K_m	the bulk modulus of matrix
K_p	the bulk modulus of particle
ΔT	temperature range measuring the CTEs
ΔT_0	the difference between the fabricating temperature and room temperature
P	pressure
r	the radius of particle
r_p	effective plastic region
t	interfacial thickness
V	volume fraction
Z	the size of the elasto-plastic zone $= r\left[\frac{(\alpha_m - \alpha_p)\Delta T_0 E_m}{(1-\nu_m)\sigma_y}\right]^{\frac{1}{3}}$
α	the linear coefficient of thermal expansion
ε	strain
ε_d	a released strain due to the plastic deformation of the matrix
σ_y	yield strength
ρ	dislocation density due to the thermal expansion mismatch
ν_m	the Poisson's ratio of matrix
ν_p	the Poisson's ratio of particle

Subscripts

i	interface
m	a constrained matrix
m′	an unconstrained matrix
p	ceramic particle
t	an entire composite

1. Introduction

In many industrial applications [1–6], one of the important properties of a material is the coefficient of thermal expansion (CTE) related with the dimensional stability of the material under a dynamic elevation, descent, or hysteresis of temperature. However thermal expansion behaviors in alloys or composites have not yet been well understood from a viewpoint of macroscopic or microscopic metallurgical phenomena.

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The effects of the particle size on the CTEs of alloys and composites have been reported recently [7–9]. It has been demonstrated that a composite with a smaller particle has a lower linear CTE. The Al/SiC$_p$ composite fabricated by S. Elomari and co-workers [7] were shown to decrease in the CTEs with decreasing the particle size. It was suggested that the effect of the particle size on the CTEs be due to silicon oxides with a very low CTE formed on the surface layers of SiC particles. Also, in the hypereutectic Al-Si alloy fabricated using various fabrication techniques [8] it was shown that the CTEs of alloys decreased with decreasing the size of the crystallized Si phase. Here, it must not be influenced by oxidation of Si because the Si phases are crystallized in the melt of the Al alloy for solidification. As a similar effect, Ma and co-workers [9] observed that the size of the particle had a small effect on the CTE of the 2024 Al/SiC$_p$ composite.

To analyze the thermal expansion behaviors of alloys or composites, several theoretical models have been derived in many literatures [10–17], and their simple equations have been used to predict the CTEs of composites and to design composites with low CTEs. Unfortunately, however, the results predicted by these approaches are poorly matched to the experimental results obtained for practical composites. It seems that the earlier models did not consider various metallurgical factors such as the sizes and shapes of reinforcements or interfacial conditions between constituents.

The CTEs of composites must be influenced by interfacial conditions, i.e. tightly bonded, debonded, coated, or oxided states between constituents. In Yih and Chung's investigation [18], it was shown that the composite reinforced with the Cu-coated SiC whisker had a lower linear CTE than the corresponding composites with the uncoated one.

Also, since the interface between the reinforcement and the matrix in a composite is considered to be incoherent, it is expected that dislocations be generated due to the thermal mismatch between them during fabrication. The degree of dislocation generation may depend considerably on the size of reinforced particles. Recently, from studies on Al/SiC composites, Arsenault and coworkers [19] showed that dislocations were generated from the interfaces between the Al matrix and SiC particles as a result of the stress induced by a high thermal mismatch during cooling.

During the cooling of composite, residual stresses can be introduced into each constituent due to a high thermal mismatch between a ceramic particle and a metal matrix. These stresses can be also released by creating a plastic deformation zone in the metal matrix around the particle. The plastic deformation due to the high thermal mismatch has been analyzed via a continuum mechanics model [20–22] as well as a dislocation loop-punching model [23–25]. Also, Lee and co-workers [20] proposed that a particle size-dependent yield stress could be deduced from the Ashelby-Johnson model [24] for dislocation nucleation.

The purpose of this study is to understand the thermal expansion behavior of the Al-Si/SiC$_p$ composite. Thus, the primary aim of this study is to develop a new model for predicting the linear CTE of the composite with mono-dispersed spherical particles on the basis of Ashelby's cutting and welding process [26, 27]. In order to develop the theoretical model it will be considered that relaxation of residual stresses during cooling can create an elasto-plastic deformation zone around a SiC or Al_2O_3 particle in the matrix. The size of the reinforced particles and other metallurgical factors of matrix alloys and composites will also be considered.

2. Theoretical approach

2.1. Basic assumptions

To derive a mathematical expression to predict the linear CTE of Al-Si alloys with two phases or Al-SiC$_p$ composites with mono-dispersed particles, the following basic assumptions were made.

1. Each SiC or Si particle is treated as an elastic sphere embedded in an infinite soft matrix, resulting in an axially symmetrical stress distribution around the particles.
2. There is no chemical reaction between phases at the fabricating temperature of alloys or composites. The interface between the matrix and the reinforced phase is in a tightly bonded state.
3. In order to avoid the complexity of model, in the first place, interfacial problems between the matrix and the reinforced phase will not be considered. These problems will be considered later.
4. The reinforced hard particles are uniformly distributed with a mono-size in the soft matrix, and the chemical composition of the matrix is homogeneous.
5. Composites or alloys are assumed to be in a metallurgical stable state. No physical or chemical change in structure occurs during exposure to an elevated temperature or thermal cycles. The influence of the phase change is not included in this analysis.
6. No temperature gradient is present throughout the entire composites or alloys.

2.2. Development of CTE model

When the Al/SiC$_p$ composite is cooled or heated to a proper temperature, an Al matrix in the vicinity of the interface between the Al matrix and SiC particles may have entirely different physical, mechanical, and thermal properties from the Al matrix far away from the interface. It is due to the high thermal expansion mismatch between them. If it is given by interfacial characteristics of tight bonding and high thermal mismatch between the particle and the matrix during cooling, a considerable residual stress would be generated at room temperature. Therefore, it may be said that cooling will induce constrains in the matrix in the vicinity of a particle. But the matrix far away from the particle may not be influenced by the ceramic particle, and for cooling must be constrained only by the thermal extraction for itself. Therefore, we will assume that the matrix in the vicinity of the particle is a constrained matrix, and the one far away from the particle is an unconstrained matrix.

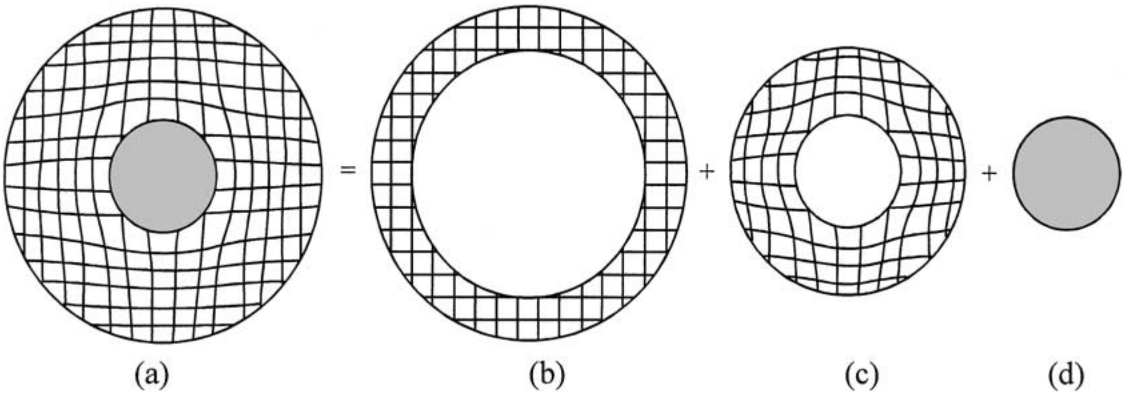

Figure 1 The composite separated into each region at the final temperature: (a) the entire composite (b) the elastic matrix region (c) the elasto-plastic matrix region (d) the elastic ceramic particle region.

In view of internal energy elastic and plastic strain energies must be distributed in each direction across the interface. Namely, the metal matrix in the vicinity of a spherical ceramic particle has both elastic and plastic strain energies [28–31], but the unconstrained matrix has only an elastic strain field. It can be regarded an entire composite as the sum of an elastic region in the unconstrained matrix, an elasto-plastic region in the constrained matrix, a high elastic region in the ceramic particle, and an interfacial region between the matrix and the particle. However, according to our third assumption, the influence of an interfacial region will be considered later. Consequently, an approach to analyze the CTE of composite can be made by extracting and reinserting these regions as shown in Fig. 1. Fig. 1 shows that the composite can be separated into each region. Fig. 1a–d represent the unconstrained matrix, the constrained matrix, and the unconstrained particle, constrained particle respectively. This new approach has been made on the basis of the Ashelby's cutting and welding process [26, 27]. In their approach only an interaction between the constrained matrix and the ceramic particle was considered. However, in our approach it will be considered that the matrix is subdivided into the regions with the constrained elasto-plastic matrix and the unconstrained elastic matrix.

Next, the linear CTE of the composite can be theoretically calculated by considering an interaction among each subdivided constituent. That is, the linear CTE of the entire composite will be set by the following separation:

(The CTE of the entire composite)
= (The contribution of the elastic matrix region) + (Contributions of an interaction between the elasto-plastic matrix region and the particle region with the higher elastic property).

Therefore, the separated constituents in the composites may be able to be set independent, and the CTEs of the entire composite can be expressed by introducing the rule of mixtures, as shown in the following equation:

$$\alpha_{\mathrm{t}} = \alpha_{\mathrm{m'}} V_{\mathrm{m'}} + \alpha_{(\mathrm{p}\Leftrightarrow\mathrm{m})} V_{(\mathrm{p+m})} \qquad (1)$$

where the identification for all symbols and subscripts will be shown in Nomenclature later. Each volume fraction of the constituents can be expressed by $V_{\mathrm{total}} = V_{\mathrm{m'}} + V_{(\mathrm{p+m})} = 1$.

$$V_{\mathrm{m'}} = 1 - (V_{\mathrm{m}} + V_{\mathrm{p}}),\ V_{\mathrm{m}} = \left[\left(\frac{Z_{\mathrm{p}}}{r}\right)^3 - 1\right] V_{\mathrm{p}} \qquad (2)$$

where Z_{p} represents the size of the elasto-plastic zone created by the thermal expansion mismatch between the soft matrix and a hard particle during cooling of the composite, and it was reported by Mortensen *et al.* [29] and Kim *et al.* [28].

The elasto-plastic matrix can be separated from the elastic matrix by introducing an elasto-plastic zone in the entire matrix of the composite. Therefore, first, the linear CTE can be calculated by considering the interaction between an elasto-plastic matrix and a ceramic particle. Next, the interaction between the elasto-plastic matrix around the ceramic particle and the unconstrained elastic matrix around the elasto-plastic matrix can be considered. Since the elasto-plastic matrix contains both the elastic and the plastic strain energy, in order to develop a CTE model these properties should be considered. Assuming that the elasto-plastic region contains an effective constrained matrix, the elastic sphere should interact directly with the severely constrained inner matrix, region II-a in Fig. 2 a and indirectly with the very weakly constrained outer matrix, region II-b in Fig. 2a. In order to simplify this concept let's assume that the elastic particle and the severely constrained matrix particle with a spherical shape, are embedded in the very weakly constrained elastic matrix as shown in Fig. 2b.

Now, it is possible to consider the interactions not only between the elasto-plastic matrix and the ceramic particle but also between the elastic matrix and the plastic matrix in the elasto-plastic region. If the composite fabricated at a high temperature is heated again to a certain high temperature, the ceramic particle and the matrix will be expanded with the other constrained. Arriving at the final temperature, the interface between them must be located at an equilibrium point ($x = c$) as a result of the interaction between these two spherical particles and the influence of the surrounding matrix as shown in the schematic diagram of Fig. 3.

For a decrement of temperature, $\mathrm{d}T$, the thermal strain of a material is expressed by $\alpha \mathrm{d}T$, where α is the linear CTE. This thermal strain occurs without any applied stress. The ceramic particle and the matrix would be strained by $\alpha_{\mathrm{p}}\mathrm{d}T$ and $\alpha_{\mathrm{m}}\mathrm{d}T$ for the same

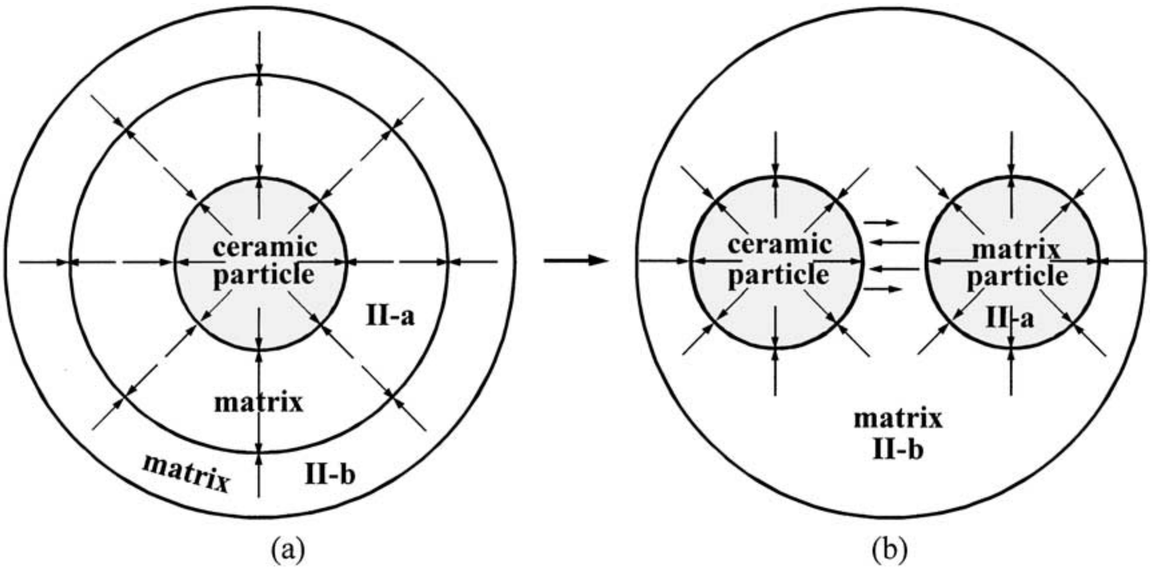

Figure 2 Schematic diagrams showing the interactions among the ceramic particle, the particle matrix, and the elastic matrix in the elasto-plastic zone. (a) an original state (b) a modified concept.

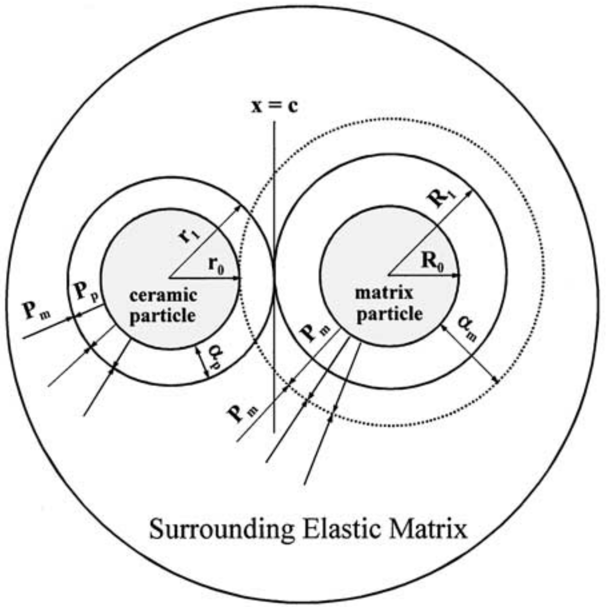

Figure 3 A schematic diagram showing an interaction between the elastic ceramic particle and a plastic matrix particle in the surrounding elastic matrix.

temperature change, but in general the quantity of these strains would be unequal. Another strain may be created due to the relaxation of the stress during cooling. This strain ε_{d} can be expressed by introducing creation of dislocations. If ε_{p} and ε_{m} denote the unconstrained states in the ceramic particle and the matrix, respectively, these strains can be expressed as follows:

$$\varepsilon_{\mathrm{p}} = \alpha_{\mathrm{p}} \Delta T \tag{3}$$

$$\varepsilon_{\mathrm{m}'} = \varepsilon_{\mathrm{m}} + \varepsilon_{d} = \alpha_{\mathrm{m}} \Delta T \tag{4}$$

To make an expression for the plastic strain in the elasto-plastic strained matrix, a relationship between a dislocation density and a strength increment will be introduced. Namely, on the basis of the dislocation strengthening theory [32], the increment of the yield strength due to the dislocation generation is given by $\Delta\sigma_{\mathrm{y}} = C \cdot G \cdot b \cdot \sqrt{\rho}$. Also, the stress relief process due to the thermal mismatch may be involved in the generation of prismatic loops or tangled dislocations [31]. In case of well-bonded composite relaxation of the residual stress due to the big thermal mismatch during cooling can be disclosed by the plastic deformation zone in the soft metal matrix around a ceramic particle. Accordingly, the interaction between them should be considered by introducing the elasto-plastic zone. At room temperature, the strain distribution in the matrix must be modified by a stress increment due to the high dislocation density near the interface. Hence, to ensure the required continuity for the interaction between them, a released strain ε_{d} due to the plastic deformation of the matrix must be superimposed on the thermal strain. Therefore, Equation 4 the expression for the strain in the matrix due to thermal mismatch can be replaced by

$$\begin{aligned}\varepsilon_{\mathrm{m}} &= \alpha_{\mathrm{m}} \Delta T - \varepsilon_{\mathrm{d}} = \alpha_{\mathrm{m}} \Delta T - \frac{\Delta\sigma_{\mathrm{y}}}{G_{\mathrm{m}}} \\ &= \alpha_{\mathrm{m}} \Delta T - C \cdot b \cdot \sqrt{\frac{(\alpha_{\mathrm{m}} - \alpha_{\mathrm{p}}) \Delta T_0 6\sqrt{2}}{b(Z^3 - r^3)/r^2}}\end{aligned} \tag{5}$$

Now, it is possible to consider a situation that the ceramic particle and the matrix with a spherical shape are expanded with the elevation of the temperature in Fig. 3. If one raises the temperature in such a situation as shown in Fig. 3, equilibrium state such as at $x = c$ will be established for a final boundary between the hard particle and the matrix particle. From the viewpoint of the plastic matrix particle located on the right hand side in Fig. 3, the solid lines represent the final displacement constrained by the elastic ceramic particle and the surrounding elastic matrix simultaneously. Details for the evolved sequence of equations are described in Appendix.

Finally, considering the equilibrium state of the total strain modified by the volume fraction of each constituent in the composite at a distance of $x = x$, the relationship that $\varepsilon_{\mathrm{m}} V_{\mathrm{m}}$ is equal to $-\varepsilon_{\mathrm{p}} V_{\mathrm{p}}$ will be obtained. Thus, the linear CTE in the region with the elastic ceramic particle, the plastic matrix particle, and the surrounding elastic matrix can be expressed by each equation. Consequently the linear CTE of the entire composite of Equation 1 can be rewritten as follows:

$$\alpha_{\mathrm{t}} = \alpha_{\mathrm{m}'} V_{\mathrm{m}'} + \alpha_{(\mathrm{p}\Leftrightarrow\mathrm{m})} V_{(\mathrm{p+m})} = \alpha_{\mathrm{m}} \cdot V_{\mathrm{m}'} + \frac{A \cdot V_{\mathrm{p}} \cdot \alpha_{\mathrm{p}} + B \cdot V_{\mathrm{m}} \cdot \alpha_{\mathrm{m}} - \dfrac{B \cdot V_{\mathrm{m}} \cdot C \cdot b}{\Delta T} \sqrt{\dfrac{(\alpha_{\mathrm{m}} - \alpha_{\mathrm{p}}) \cdot \Delta T_0 \cdot 6\sqrt{2}}{b \cdot (Z^3 - r^3)/r^2}}}{A \cdot V_{\mathrm{p}} + B \cdot V_{\mathrm{m}}} \tag{6}$$

where

$$A = \frac{4a^3 G_{\mathrm{m}}}{3K_{\mathrm{p}} + 4a^3 G_{\mathrm{m}}}, \qquad B = \frac{4G_{\mathrm{m}}}{3K_{\mathrm{m}} + 4G_{\mathrm{m}}},$$

and

$$Z = r \cdot \sqrt[3]{\frac{(\alpha_{\mathrm{m}} - \alpha_{\mathrm{p}}) \Delta T_0 E_{\mathrm{m}}}{(1 - \nu_{\mathrm{m}}) \cdot \sigma_{\mathrm{y}}}}.$$

The developed CTE model represented by Equation 6 is distinguished from the previous models. In this model the interaction between the hard particle and the soft matrix during cooling was considered by introducing the concepts of the plastic property of the matrix and their contributions. Note that the particle volume fraction, Young's modulus and Poisson's ratio as well as the plastic zone and the particle size may affect the linear CTE of the composite.

However, despite the development of the new CTE model considered by introducing the elasto-plastic zone, the interfacial characteristics between the ceramic particle and the metal matrix was not considered in this model. Also the variation of the CTE of the composite with the particle size should be revealed later.

2.3. Extension of the model

No interfacial effect on the CTE of the composite has been applied as yet; we have considered only the interaction between the elastic particle and the matrix arising from the thermal mismatch between them during cooling. Maybe we can consider the effects of interfaces on the CTE of the homogeneous composite through the extracting and inserting procedure. The apparent density of a composite with a higher volume fraction, a smaller particle, and more phases, may be smaller than the theoretical one. Since it seems to be related to the introduction of defects such as phase boundaries and dislocations, probably a relative interfacial effect may be more important. Therefore, Equation 6 should be replaced by the following equation in which an interfacial effect is considered by the rule of mixtures through the extracting and inserting procedure.

$$\alpha_{\mathrm{t}} = \alpha_{\mathrm{m}} V_{\mathrm{m}'} + \alpha_{(\mathrm{p}\Leftrightarrow\mathrm{m})} V_{(\mathrm{p+m})} \pm \alpha_{\mathrm{i}} V_{\mathrm{i}} \tag{7}$$

where the third term, $\alpha_{\mathrm{i}} V_{\mathrm{i}}$, indicates the influence of an interfacial region, and the positive or negative sign is determined by the interfacial characteristics. If the interfacial region plays an important role of a sink for thermal expansion, the sign should be negative. For example, the apparent linear CTE of composite with the looser phase boundaries may be measured by the lower value than that of composite with the tighter phase boundaries. On the other hand, the positive sign may apply to the composite with a chemical reaction between the matrix and the particle resulting in the formation of an intermetallic compound with a relatively compact lattice structure [34]. Since the chemical reaction between them decreases the volume of the matrix and the reinforcement, their effective volume may be determined by chemical elements in the reaction zone. Also if cooling is performed, a crack may be initiated due to the large thermal mismatch in this reaction zone with a brittle phase. Because this chemical reaction zone may absorb the thermal stress due to cooling, the elasto-plastic region in the matrix around the reinforcement must rarely form.

Because the linear CTE, α_{i}, of the interfacial region may differ from those of the matrix and the particle, it is very difficult to consider this problem. Assuming that the interfacial region may play an important role of a sink for the thermal expansion, it will be reasonable to subtract the volumetric quantity of the interfacial region from the volume calculated by considering the entire composite ignoring the region.

3. Results and discussions: Verifying the developed CTE model

To verify the validity of the newly developed model, the CTEs for Al-Si binary alloys were calculated for various sizes and weight percents of the crystallized Si particles in the temperature range from 293 K to 573 K on the basis of Equation 6, and then compared with the previous experimental results. The calculated and experimental results are shown in Fig. 4. Table I shows the metallurgical input parameters and variables used in calculations from Figs 4 to 7. The symbols in Fig. 4 represent the linear CTE measured by Thermo-Mechanical Analyzer for the hypereutectic Al-Si industrial alloys (JIS A390) that have been fabricated by various processing technologies followed by the T6 heat treatment [8]. The lines in Fig. 4 represent the linear CTEs calculated by the developed model for the Al-x wt.%Si binary alloys. It shows that the theoretically calculated results are slightly deviated from those measured experimentally. These deviations may be owing to the assumptions such as the spherical shape, no interfacial volume, and no precipitation effect. Despite these

TABLE I Input data of metallurgical parameters for calculating the CTE of Al alloys and composites

Parameters	Al(pure)	Al(6061)	Si	SiC_p	Al_2O_3
The linear CTE (α, $\times 10^{-6}$ μm/μmK)	25.3	25.4	4.1	4.7	7.2
Bulk modulus (K, GPa)	69	78.9	93.3	227.3	270.3
Shear modulus (G, GPa)	26	26.5	—	—	—
Poisson's ratio (ν)	0.33		0.20	0.17	0.25
Yield strength (σ_y, MPa)	28	145	—	—	—
Burger's vector (μm)	0.283×10^{-3}		—	—	—
Particle diameter (μm)	—		3.6, 6, 11, 12	10, 20, 40	12, 26
Constant (C)	1.25		—	—	—

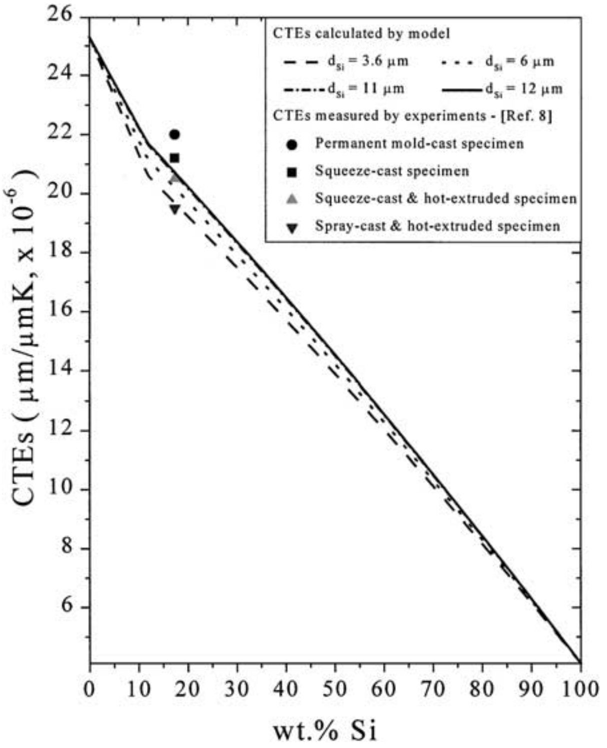

Figure 4 Calculated and measured CTEs of Al-x wt.%Si alloys.

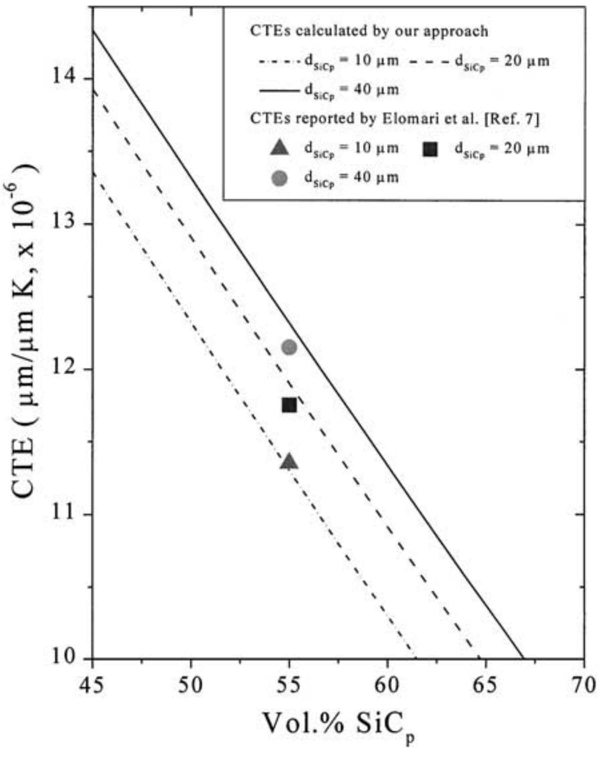

Figure 5 Calculated and reported CTEs of pure-Al/SiC_p composites.

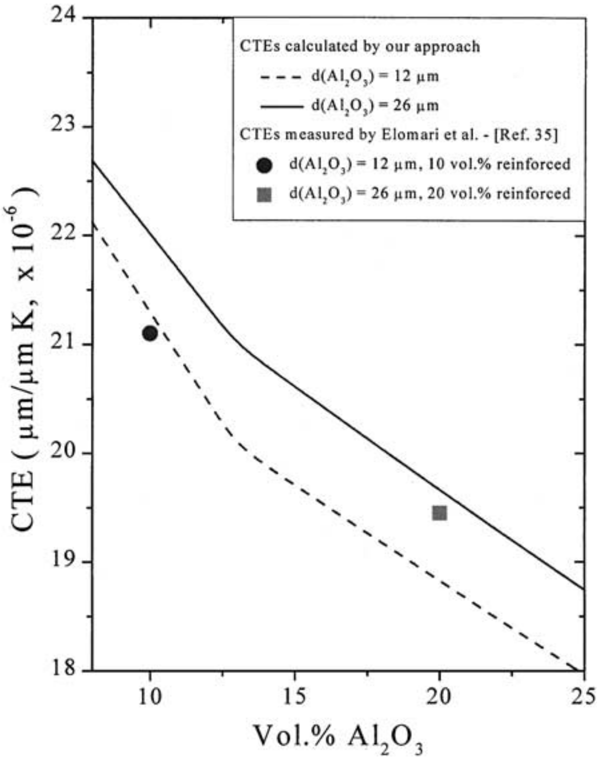

Figure 6 Calculated and reported CTEs of 6061Al/Al_2O_3 composites in the temperature range from 298 K to 673 K.

deviations, it seems that the calculated CTEs agree well with the measured values.

To verify the validity of our extended model, the calculated CTEs have been compared with the reported CTEs for binary composite. S. Elomari and others [7] have measured the CTEs for the pure-Al/SiC_p composite fabricated by the melt infiltration technique. In Fig. 5 the symbols represent the experimental values and the lines represents the one calculated theoretically using our extended model. Fig. 5 shows that the linear CTEs calculated by our extended model agree pretty well with their experimental values. Therefore, it seems that the linear CTE of the composite with two phases is also influenced by interfacial characteristics.

In order to satisfy the situation that the reinforcement and the matrix are replaced by that with different properties, it should also be investigated whether the calculated CTEs agree with the experimental results or not. Elomari and others [35] have reported the CTEs measured for the rolled sheet of the 6061Al/Al_2O_3 composite. In Fig. 6, the symbol represents the linear CTEs measured experimentally by them for 10 vol.% and 20 vol.% Al_2O_3 reinforced composites, respectively, and the lines show the linear CTEs calculated on the basis of our developed model.

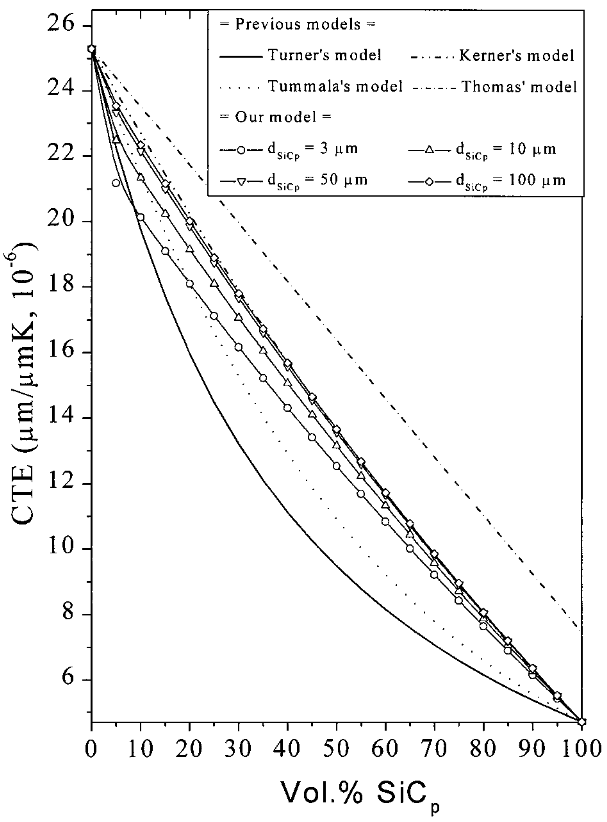

Figure 7 Comparison between the developed CTE model and previous models for predicting the CTE of the Al/SiC_p composite with two phases in the temperature range from 298 K to 573 K.

Also, to understand the difference between the newly developed model and the previous ones will be helpful in verifying the validity of our CTE model. Fig. 7 shows the results calculated on the basis of the models proposed by previous investigators [10–14] and that developed in this study. Note that the size effect of the reinforcement on the linear CTEs is taken into consideration in our model. From the results shown in Fig. 7, it appears that the linear CTE of the composite with two phases depends upon the size of the reinforcement although previous models cannot predict this effect. This comparison suggests that the effect of the secondary phase with different sizes on the linear CTE can be considered and predicted in the new model.

4. Conclusions

To analyze the thermal expansion behaviors of Al-Si alloys and composites, a new approach for the CTE on the basis of Ashelby's cutting and welding process was made. In this theoretical approach, it was considered that relaxation of residual stress could create an elasto-plastic zone in the matrix around a particle during cooling. To simplify the development of the model, an effective deformed area in the elasto-plastic zone was assumed to be a spherical matrix particle, and it was also assumed that this soft matrix particle be directly influenced by the reinforced particle when temperature was elevated. To consider the contribution of the interfacial characteristics between them, a new extended CTE model was also proposed, and it was verified by comparing the calculated values based on the new model with the experimental results reported by previous workers. We may conclude as follows:

1. A new theoretical approach for analyzing the thermal expansion behaviors of alloys and composites with two phases has been developed, an effect of the particle size on the linear CTE is considered in this approach.
2. In an extended approach, it has been suggested that the linear CTEs of alloys and composites are also influenced by the interfacial characteristics between the metal matrix and the reinforcement.
3. This CTE model reveals that the linear CTE of a composite with two phases depends upon the size of the reinforcement whereas previous models can not predict this effect.
4. The CTEs of material systems with two phases depend on various metallurgical parameters such as the size of elasto-plastic zone, the size and shape of constituent, and the interfacial characteristics as well as the volume fraction of constituent, the elastic modulus, and Poisson's ratio.

Appendix

According to our assumptions, let's consider a situation that the ceramic particle and the matrix with a spherical shape are expanded with the elevation of the temperature in Fig. 3. Assuming linear thermal expansion and no shear stresses acting on these particles, the total strain would be simplified by $\varepsilon = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \varepsilon_{11}$ since $\varepsilon_{22} = \varepsilon_{33} = 0$. Also, the ceramic particle and the matrix with a spherical shape will be constrained by the elastic matrix around these particles. Because these particles are in uniform hydrostatic compression, the total stress acting on each particle can be expressed by $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma$. At the distance of x, therefore, the strains acting on each particle can be expressed as

$$\varepsilon_{\mathrm{p}(x)} = \frac{\sigma_{11} - \nu_{\mathrm{p}}(\sigma_{22} + \sigma_{33})}{E_{\mathrm{p}}} = \frac{(1 - 2\nu_{\mathrm{p}})}{E_{\mathrm{p}}} \cdot \sigma_{\mathrm{p}} \quad (1)$$

$$\varepsilon_{\mathrm{m}(x)} = \frac{\sigma_{11} - \nu_{\mathrm{m}}(\sigma_{22} + \sigma_{33})}{E_{\mathrm{m}}} = \frac{(1 - 2\nu_{\mathrm{m}})}{E_{\mathrm{m}}} \cdot \sigma_{\mathrm{m}} \quad (2)$$

On the other hand, during the elevation of temperature, the stresses constrained by the outer elastic matrix can be considered by terms of pressure as follows:

$$\varepsilon_{\mathrm{p}(x)} = \frac{\sigma_{11} - \nu_{\mathrm{p}}(\sigma_{22} + \sigma_{33})}{E_{\mathrm{p}}} = \frac{(1 - 2\nu_{\mathrm{p}})}{E_{\mathrm{p}}} \cdot (-P_{\mathrm{p}}) \quad (3)$$

$$\varepsilon_{\mathrm{m}(x)} = \frac{\sigma_{11} - \nu_{\mathrm{m}}(\sigma_{22} + \sigma_{33})}{E_{\mathrm{m}}} = \frac{(1 - 2\nu_{\mathrm{m}})}{E_{\mathrm{m}}} \cdot (-P_{\mathrm{m}}) \quad (4)$$

In order to get P_{p} and P_{m}, interactions among the elastic ceramic particle, the plastic matrix particle, and the surrounding elastic matrix should be considered. Assuming that the plastic property in the surrounding elastic matrix can be ignored, then each particle in an equilibrium state will behave like under the hydrostatic pressure. At $x = c$, hence, the displacement by expansion of each particle due to the elevation of temperature should be expressed as follows:

1. In case of the elastic ceramic particle with a spherical shape

$$\varepsilon_{\mathrm{p}(x=c)} = \frac{(1-2\nu_{\mathrm{p}})}{E_{\mathrm{p}}} \cdot (-P_{\mathrm{p}}) + \alpha_{\mathrm{p}} \cdot \Delta T \qquad (5)$$

2. In case of the plastic matrix particle with a spherical shape

$$\varepsilon_{\mathrm{m}(x=c)} = \frac{(1-2\nu_{\mathrm{m}})}{E_{\mathrm{m}}} \cdot (-P_{\mathrm{m}}) + \alpha_{\mathrm{m}} \cdot \Delta T - C \cdot b \cdot \sqrt{\rho} \qquad (6)$$

3. In case of the surrounding elastic matrix particle

If temperature is elevated, the constrained state, acting on the surface of each spherical particle at $x = c$, should be the principal stress state of isotropic tension ($\sigma_{11} = 2\sigma_{22} = 2\sigma_{33}$). Assuming that the plastic property of the matrix around each particle can be ignored and that the size of the ceramic particle and the matrix particle differ from each other, then a pressure effect will be created due to the size difference. Again assuming the relationship of $r_{\mathrm{p}} = a \cdot r$ at $x = c$ is satisfied, the strain acting on each particle can be expressed by the following equations. This concept has been developed by Tummala and Friedberg [12] using Timoshenko's analysis [33] which relates the principal stresses to the pressure on the matrix due to the spherical particle.

A. In the surrounding elastic matrix interacting with the elastic ceramic particle

$$\varepsilon_{\mathrm{m}'(x=c)} = \frac{1+\nu_{\mathrm{m}}}{2E_{\mathrm{m}}} \cdot \left(\frac{1}{a}\right)^3 \cdot P_{\mathrm{p}} + \alpha_c \cdot \Delta T \qquad (7)$$

B. In the surrounding elastic matrix interacting with the plastic matrix particle

$$\varepsilon_{\mathrm{m}'(x=c)} = \frac{1+\nu_{\mathrm{m}}}{2E_{\mathrm{m}}} \cdot (P_{\mathrm{m}}) + \alpha_c \cdot \Delta T \qquad (8)$$

In the composite, the above strain relationships must be satisfied at the interfaces of the surrounding elastic matrix, the elastic ceramic particle, and the plastic matrix particle. For a composite with two phases, therefore, the pressure acting on each particle may be determined as follows, assuming that strains have continuity across the surrounding elastic matrix and particles.

1. Equation 5 = Equation 7;

$$\varepsilon_{\mathrm{p}(x=c)} = \varepsilon_{\mathrm{m}'(x=c)};\ P_{\mathrm{p}} = P_{\mathrm{m}'} \qquad (9)$$

2. Equation 6 = Equation 8;

$$\varepsilon_{\mathrm{m}(x=c)} = \varepsilon_{\mathrm{m}'(x=c)};\ P_{\mathrm{m}} = P_{\mathrm{m}'} \qquad (10)$$

In order to get an expression of strains acting on each particle as it is expanded to $x = x$, P_{p} and P_{m} of Equations 9 and 10 must be inserted into Equations 3 and 4, respectively. Then, Equations 3 and 4 must be rewritten as follows:

$$\varepsilon_{\mathrm{p}(x=x)} = \frac{(1-2\nu_{\mathrm{p}})}{E_{\mathrm{p}}} \cdot (-P_{\mathrm{p}}) = \frac{(1-2\nu_{\mathrm{p}})}{E_{\mathrm{p}}} \times \frac{(\alpha_c - \alpha_{\mathrm{p}}) \cdot \Delta T}{\left(\frac{1}{a}\right)^3 \cdot \frac{(1+\nu_{\mathrm{m}})}{2E_{\mathrm{m}}} + \frac{(1-2\nu_{\mathrm{p}})}{E_{\mathrm{p}}}} \qquad (11)$$

$$\varepsilon_{\mathrm{m}(x=x)} = \frac{(1-2\nu_{\mathrm{m}})}{E_{\mathrm{m}}} \cdot (-P_{\mathrm{m}}) = \frac{(1-2\nu_{\mathrm{m}})}{E_{\mathrm{m}}} \times \frac{(\alpha_c - \alpha_{\mathrm{m}}) \cdot \Delta T + C \cdot b \cdot \sqrt{\rho}}{\frac{(1+\nu_{\mathrm{m}})}{2E_{\mathrm{m}}} + \frac{(1-2\nu_{\mathrm{m}})}{E_{\mathrm{m}}}} \qquad (12)$$

Considering the equilibrium state of the total strain modified by the volume fraction of each constituent in the composite at a distance of $x = x$, the relationship that $\varepsilon_{\mathrm{m}} V_{\mathrm{m}}$ is equal to $-\varepsilon_{\mathrm{p}} V_{\mathrm{p}}$ will be obtained. Therefore, the linear CTE in the region with the elastic ceramic particle, the plastic matrix particle, and the surrounding elastic matrix can be obtained. Finally, the linear CTE of the entire composite can be rewritten as Equation 6 in the main text.

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